Bingham Plastic Fluid Model for Steady Flow of Blood with Velocity Slip Tube Wall in Presence of Magnetic Field

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Abstract: **-In this study, the effect of magnetic field on poiseuille flow of Bingham plastic fluid model for blood with velocity slip and no slip is examined. In this modeling, an interaction of non-Newtonian nature of blood and its flow through arteries (aorta, femoral, carotid, coronary and arteriole), in presence of wall slip has been attempted. Effort has been made to indicate the behavior of flow variation with Hartmann number. The application of Magneto dynamics in physiological flow problem is of growing interest. The flow of blood can be controlled by applying magnetic field. Mathematical modeling for poiseuille flow of blood (-a Bingham plastic fluid model) with an axial velocity slip along an artery wall in presence of magnetic field, is considered. It is observed that when Hartmann number increases the fluid velocity is greatly affected. The present model includes the poiseuille flow models of slip and no slip at artery wall and one- layered Bingham plastic fluid model with zero-slip, as its special cases. Applications of this theoretical modeling to cardiovascular diseases and the role of slip in the better functioning of the diseased or occluded arteries are included in brief.**

Keywords: **Magnetic field, Reynolds number, Hartmann number Bingham Plastic Fluid Model, Blood flow**

I. INTRODUCTION

Blood flow through arteries can be complicated by the formation of atherosclerotic plaque the artery wall its subsequent advancement impedes the flow through and artery. This unwanted growth at vessel wall may ultimately affect the wall shear stress distribution [14]. The cardiovascular system of man and animals is characteristically a branch network of distensible tubes which carry blood from the heart to periphery and back again [19][4][6] .The primary function of circulation is to transport nutrient to tissue and to remove metabolic product ,Human blood is a suspension of red cells in a continuous and aqueous substance called plasma[10][11]. The plasma behaves like a Newtonian fluid with a co-efficient of viscosity 1.2 centipoises whereas the whole blood is shear dependent that is the apparent viscosity of whole blood decreases with an increase rate of shear it has a infinite yield stress under certain flow condition and the viscosity of blood varies with hematocrit and also changes with temperature as well as dieses state .At high shear rates blood behaves as a Newtonian fluid with constant viscosity in larger arteries ,diameter nearly above 1 mm as shear stress decreases blood shows a Non Newtonian character [17] and other [13][16],have pointed out that viscosity of blood in general and interior viscosity of red cells in particular can be in significant factor in pathogenesis of ischemia and infraction and may play an important role in hypertension and cardiovascular disease[18]. In most of the theoretical models on blood flow, usual no slip condition at vessel wall is considered[2][3][5][1] have suggested the likely presence of a red cell slip at vessel wall or in its immediate neighborhood and in view of a possible existence of slip at tube wall.[15][12] and other have considered a velocity slip condition at blood vessel wall or at interface of fluid in their modeling, in the present modeling the blood flow through an artery a slip condition for velocity at tube wall of five different locations of CVS is employed.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

For one dimensional flow axial velocity $\hat{v}$ = (0, 0, $u_z(r)$) the equation for steady tube flow $0 \le r \le R$ of blood (a-Bingham fluid) in (r,θ,z) co-ordinate system reduce to the following form in presence of transverse magnetic effect.

$$-\frac{\partial \hat{p}}{\partial \hat{r}} = 0 \qquad (1)$$

$$-\frac{1}{\hat{r}}\frac{\partial \hat{p}}{\partial \hat{\theta}} = 0 \qquad (2)$$

$$-\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{z}} + \frac{\mu}{\hat{r}}\frac{1}{\rho}\frac{\partial}{\partial \hat{r}}\left(\hat{r}\frac{\partial \hat{u}_z}{\partial \hat{r}}\right) + \frac{\sigma}{\rho}\hat{B}^2 u = 0 \qquad (3)$$

From which we observed that pressure does not vary in the radial $(\hat{(r)})$,circumferential ($\hat{\theta}$) and axial $(\hat{z})$ direction and that pressure remain constant across any cross-section of the tube and $\hat{p}$ Is a function of only $(\hat{z})$ that is $\hat{p} = p(\hat{z})$ and so pressure gradient term in the last equation above becomes $\frac{d\hat{p}}{d\hat{z}}$

Then equation (3) becomes,

$$-\frac{1}{\rho}\frac{\partial\hat{p}}{\partial\hat{z}}+\frac{\mu}{\hat{r}}\frac{1}{\rho}\frac{\partial}{\partial\hat{r}}\left(\hat{r}\frac{\partial\hat{u}_z}{\partial\hat{r}}\right)+\frac{\sigma}{\rho}\hat{B}^2u=0 \quad (4)$$

We use the following non-dimensional quantities

$$r=\frac{\hat{r}}{R_0},z=\frac{\hat{z}}{R_0},R=\frac{\hat{R}}{R_0},P=\frac{\hat{P}}{\rho U_0{}^2},U=\frac{\hat{U}}{U_0},B=\frac{\hat{B}}{U_0} \quad (5)$$

Then equation (4) becomes,

$$C+\frac{1}{R_e r}\frac{d(r\tau_{rz})}{dr}+\frac{M^2}{R_e}=0 \quad (6)$$

Where

$$C=\frac{\rho U_0{}^2}{R_0}\frac{dp}{dz},M=\sqrt{\frac{\sigma}{\rho v}}BR_0,R_e=\frac{R_0U_0}{v},\tau_{rz}=\mu\frac{du_z}{dr}=\mu e.^0$$

Now we will solve the equation (4)

$$r^2\frac{d^2u_Z}{dr^2}+\frac{du_Z}{dr}=Kr^2 \text{ Where } K=-\frac{C}{\mu}R_e r-\frac{M^2r}{\mu}$$

Let $r=e^z$, $D=\frac{d}{dz}$

Solution:

$$U_z=\frac{1}{4}\left[-\frac{C}{\mu}R_e r-\frac{M^2r}{\mu}\right]r^2 \quad (7)$$

Again, shear stress component at any distance r from the tube axis is given by

$$\tau_{rz}=\mu\frac{du_z}{dr}=\mu e^0 \quad (8)$$

i.e.,

$$\tau_{rz}=\mu\frac{r}{2}\left[-\frac{C}{\mu}R_e-\frac{M^2}{\mu}\right] \quad (9)$$

Express for wall shear stress τ_w can be obtained from the formula

$$\tau_w=\tau_{rz}(r=R) \quad (10)$$

$$\Rightarrow \tau_w=-\frac{R}{2}\left[CR_e+M^2\right] \quad (11)$$

Using Equation (7) and $\tau_Z(r_0)=\tau_0$ express for τ_0 will lead to the form

$$\tau_0=-\frac{r_0}{2}\left[CR_e+M^2\right] \quad (12)$$

In between τ_0 and τ_w there may arises two cases wall shear stress is greater and that yield stress. In case $\tau_0\rangle\ \tau_w$ that is if $r_0\rangle R$ then there will occur no flow accordingly velocity function will become

$$U_Z=\frac{r^2}{4}\left[-\frac{C}{\mu}R_e-\frac{M^2}{\mu}\right] \quad (13)$$

Also Bingham equation (7-8) may be described in the following form

$$e^0=f(\tau_{rz})=\frac{1}{\mu}(\tau_{rz}-\tau_0) \qquad \tau_{rz}\geq\tau_0 \quad (14)$$

$$e^0=\frac{1}{\mu}\left[-\frac{r_0}{2}\left(CR_e+M^2\right)+\frac{r_0}{2}\left(CR_e+M^2\right)\right] \quad r=r_0$$

$$=0 \qquad \tau_{rz}\leq\tau_0 \quad (15)$$

In the above, vanishing of strain rate that is $e^0=0$ implies that

$$\frac{du_Z}{dr}$$

which after integration yield to

$$\int\frac{du_Z}{dr}dr= \text{constant}$$

U_Z =constant

U_Z =constant= U_0 When $\tau=\tau_0$ (16)

Where U_0 is the core velocity at $r=r_0$ (core radius) .As such for blood flow when $r_0\langle R$ there arises two region $0\leq r\leq r_0$ and $r_0\leq r\leq R$ and it is clear for region between 0 and r_0 equation representing the flow is

$$\frac{du_Z}{dr}=0 \quad 0\leq r\leq r_0 \quad (17)$$

Which after integration give rise to the form $U_Z=U_0 \quad 0\leq r\leq r_0$ indicating the velocity profile will become flat in the region and for $r_0\leq r\leq R$ velocity U_Z will show deviation from flat profile and Bingham equation (16) has to be applied for this domain of blood flow the same equation it is easily seen that

$$\frac{du_Z}{dr} = \left[\frac{r_0 - r}{2\mu}\left(CR_e + M^2\right)\right] \quad r_0 \le r \le R \quad (18)$$

III. SOLUTION OF THE PROBLEM

In solving equation (18) we have used the following velocity slip condition at vessels walls $u_Z = u_S$ at $r = R$ (19)

where U_S is the constant slip velocity at tube wall in axial distance .As a result of integration between r and R we have

$$\int_r^R \frac{du_Z}{dr} dr = \int_r^R \frac{r_0 - r}{2\mu}\left(CR_e + M^2\right) dr \quad (20)$$

After integrating the above equation (23) we get

$$u_z = u_s + \frac{\left(CR_e + M^2\right)(R - r)}{4\mu}\left[(R + r) - 2r_0\right]$$

$$r_0 \le r \le R \quad (21)$$

At $r_0 = r$ expression for core velocity can be obtained from equation (21)

$$u_o = u_s + \frac{\left(CR_e + M^2\right)}{4\mu}\left(R - r_0\right)^2 \quad (22)$$

And for all values of r between 0 and r_0 velocity function is $u_o = u_s \quad 0 \le r \le r_0$ (23)

Thus from above expression and consideration velocity distribution u_Z can be re-written in the following manners

$$u_z = \begin{cases} u_z(r) & r_0 \le r \le R \\ u_0 & 0 \le r \le r_0 \\ 0 & r \rangle R \end{cases} \quad (24)$$

Where $u_z(r)$ and u_0 are given in equation (21) and (22) respectively

The rate of volume flow can be found from

$$Q = \int_{r=0}^{R} 2\pi r u_z dr$$

By integration after using the equation (21), (22) and (23)

$$Q = 2\pi \int_0^r r u_0 dr + 2\pi \int_{r_0}^R r u_z dr$$

i.e., $Q = I_1 + I_2$

Now,

$$I_1 = 2\pi \int_0^{r_0} r\left\{u_s + \frac{\left(CR_e + M^2\right)}{4\mu}\left(R - r_0\right)^2\right\} dr$$

$$I_1 = 2\pi\left[u_s \frac{r_0^{\ 2}}{2} + \frac{\left(CR_e + M^2\right)}{4\mu}\left\{\frac{R^2 r_0^{\ 2}}{2} + \frac{r_0^{\ 4}}{2} - R r_0^{\ 3}\right\}\right]$$

And,

$$I_2 = 2\pi \int_{r_0}^R r\left[u_s + \frac{\left(CR_e + M^2\right)(R - r_0)}{4\mu}\left[(R + r) - 2r_0\right]\right] dr$$

$$\Rightarrow I_2 = 2\pi\left[\frac{R^2}{2}u_s - \frac{r_0^{\ 2}}{2}u_s + \frac{\left(CR_e + M^2\right)}{4\mu}\left\{\frac{R^4}{4} - \frac{R^3 r_0}{3} - \frac{R^2 r_0}{2} + R r_0^{\ 3} - \frac{5}{12} r_0^{\ 3}\right\}\right]$$

Now, $Q = I_1 + I_2$

$$Q = \pi R^2 u_s + \frac{\pi R^4}{8\mu}\left\{\left(CR_e + M^2\right) - \frac{4}{3}\left(-\frac{2\tau_0}{R}\right) + \frac{1}{3}\left(\frac{-2\tau_0}{R}\right)^4 \left(CR_e + M^2\right)^{-3}\right\} \quad (25)$$

Let $\beta = \left(-\frac{2\tau_0}{R}\right)\left(CR_e + M^2\right)^{-1}$

$$A(\beta) = \left(1 - \frac{4}{3}\beta + \frac{1}{3}\beta^4\right) \quad (26)$$

Then,

$$Q = \pi R^2 u_s + \frac{\pi R^4}{8\mu}\left(CR_e + M^2\right)A(\beta) \quad (27)$$

$$U_m = \frac{Q}{\pi R^2} = u_s + \frac{\pi R^4}{8\mu}\left(CR_e + M^2\right)A(\beta) \quad (28)$$

And expression for apparent viscosity μ_a can be found from the formula

$$\mu = \frac{\pi\left(CR_e + M^2\right)R^4}{8Q} \quad (29)$$

And using equation (26), apparent viscosity takes the following

$$\mu_a = \left[A(\beta)\frac{8u_s\mu}{\left(CR_e + M^2\right)R^2}\right]^{-1} \quad (30)$$

IV. Special Case:

From equation (24)-(27) velocity representation for poiseuille flow of blood (Newtonian fluid) can be accordingly obtained with zero slip substitution $u_s = 0$ and taking $\tau_0 = 0$ or $r_0 = 0$

The parabolic velocity profile for poiseuille flow the takes the form

$$u_z(r) = \frac{(CR_e + M^2)(R^2 - r^2)}{4\mu} \quad 0 \le r \le R \quad (31)$$

Employing an axial velocity slip at the tube wall, instead of usual no slip in velocity along the wall the velocity function for poiseuille flow will takes the form

$$u_z(r) = u_s + \frac{(CR_e + M^2)(R^2 - r^2)}{4\mu} \quad 0 \le r \le R \quad (32)$$

In the aforesaid cases velocity is maximum at the axis of the tube and expression for maximum velocity obtained from equation (31) and (32) are given by

$$u_{m1} = \frac{(CR_e + M^2)R^2}{4\mu} \quad (33)$$

$$u_{m1} = u_s + \frac{(CR_e + M^2)R^2}{4\mu} \quad (34)$$

Expression for rate of volume flow Q can be accordingly obtained for above two cases in the form

$$Q_1 = \frac{(CR_e + M^2)\pi R^4}{8\mu} \quad (35)$$

$$Q_2 = \pi R^4 u_s + Q_1 \quad (36)$$

V. Results and Discussions

In the present modeling, blood flow through a uniform rigid artery at five different locations of cardiovascular system viz., aorta, femoral, carotid, and coronary and arteriole, in presence of magnetic effect has been considered. For computation of flow variables, relevant data indicating tube radius (R) and its corresponding pressure gradient (C) [7] as well as ratio of a critical radius r0 to an artery for variations of yield stress, $\tau_0(\ge 0)$, is included in Table 2.1. Also, in this case, shear stress τ_{rz} varies linearly with radial distance r and r_0 (critical radius) and R (tube radius) are the radial coordinates corresponding to the respective yield values τ_0 and τ_w of the yield stress (τ_0) for a Bingham fluid. Blood is assumed to act like a non-Newtonian fluid possessing a finite yield stress viz., a Bingham plastic fluid. There arises two distinct cases for this fluid bearing an yield property, viz.,

If shear stress τ_{rz} at a distance is not higher than a finite yield stress, blood will not flow and

If shear stress is not lower than its yield value, blood flow will be possible. Analytic expressions for flow variables viz., velocity, flow rate, mean velocity, apparent viscosity and stresses are obtained. Velocity function, expressed in equation (), appears to be a function of radial co-ordinate ®, tube radius ®, critical radius (r0), pressure-gradient ©, Bingham fluid viscosity (μ) and slip velocity (μ_s). The present analysis includes Poiseuille flow models with the cases of velocity slip and zero-slip at tube wall and, steady one-dimensional Bingham plastic fluid model with zero-slip at tube wall as its special cases in presence of magnetic effect.

Velocity can be obtained from equation () and its variation with Hartmann number M at zero and non-zero yield stresses $\tau_0(\ge 0)$ for five different locations of cardiovascular system viz., aorta, femoral, carotid, coronary and arteriole for both cases of slip and no-slip at an artery wall is presented graphically in Figures(1..15). While doing computation, three values of yield stress characterizing zero and non-zero values of τ_0 (viz., $\tau_0 = 0, 0.04, 0.1$) [19], or, corresponding values of critical radius r_0, slip velocity $u_s = 0.1$ cm/sec [19], Bingham fluid viscosity $\mu = 2$ cm [10] and, tube radius (R), and the corresponding pressure gradient (C) (as suggested in Sud and Sekhon) [7] from table 2.1, are used. Velocity profiles for a full scale of dimensionless radial coordinate r/R from the tube axis to vessel wall clearly state that

This model include poiseuille flow models with velocity slip and zero ship at vessel wall and steady one-dimensional Bingham plastic fluid model with zero wall slip, as its special cases.

Velocity shows distinct behaviour for variation of Hartmann number M in different fluid parameter viz., yield stress $\tau_0(\ge 0)$.

Profiles are parabolic at vanishing yield stress (when $\tau_0 = 0$ or $r_0 = 0$) with the maximum velocity at tube axis and the minimum one at tube wall in all five locations of CVS.

(i) But for non-vanishing yield stress i.e., for $\tau_0 = 0.04$ and 0.10, profiles indicate distinct bluntness near the axis i.e., in core region and away from the central (core) region, velocity points out a parabolic profile.

(ii) These blunted for flat profiles in velocity $(\tau_0 > 0)$ clearly exposes the non-Newtonian nature of blood.

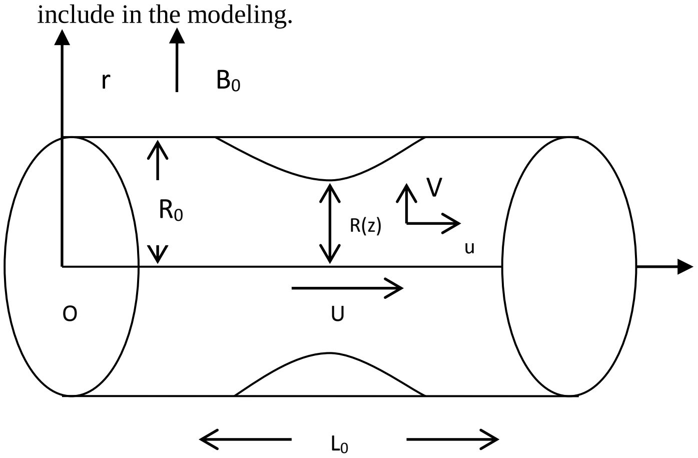
(iii) As yield stress τ_0 increases, core region increases.

(iv) As tube radius decreases (from a larger artery aorta to a smaller one coronary), core region decreases. However, core region is found to increase in arteriole than its immediate larger tube coronary.

(v) Velocity profile increases when Hartmann number M increases in different fluid parameter viz., yield stress $\tau_0(\geq 0)$. The nature of velocity profile is also same in no slip.

(vi) Analysis developed here is based on certain assumptions which may lead to some physiological implications viz., (a) Flow is assumed steady which is indeed true for very thin arteries in CVS where the pulsatility effects are small. (b) Assumption that velocity variation in axial direction is negligible as compared to its variation in radial direction, may lead to the implication that the length of the artery is too large as compared to the radius.

In this modeling, an interaction of non-Newtonian nature of blood and its flow through arteries (aorta, femoral, carotid, coronary and arteriole), in presence of wall slip has been attempted. Effort has been made to indicate the behavior of flow variation with Hartmann number M. Biswas [3], Chaturani and Biswas [19] and, Prahlad and Schultaz [9] and others have already reported that flow variables like velocity profiles, flow rate, shear stresses, apparent viscosity etc. could take an important role in the fundamental understanding and development of cardiovascular, cerebrovascular, arterial and other diseases. It is therefore essential from the physiological point of view, to propose a more appropriate model for measuring the flow variables more accurately. In the analysis, a constant slip and its arbitrary value as well as arbitrary variation of fluid property n are taken. It is observed that when Hartmann number increases the fluid velocity is greatly affected. The mathematical expressions may help medical practitioners to control the blood flow of a patient whose blood pressure is very high, by applying certain magnetic field. This analysis could be improved if an appropriate measure of velocity slip and fluid property, in accordance with cell concentration and artery diameter, are include in the modeling.



VI. CONCLUSION

In this paper we have attempted to study the behaviour of poiseuille flow of Bingham plastic fluid model for blood flow with velocity in presence of magnetic effect. A steady one-dimensional flow of blood (-a Bingham fluid) subject to the boundary conditions of velocity slip, suggested in the models of Biswas [3], Chaturani and Biswas [19] and, Prahlad and Schultaz [9], for five different locations of CVS, in presence of magnetic effect is investigated. Analytic expressions for velocity, flow rate, shear stress at wall, yield stress and apparent viscosity are presented. Axial velocity appears to be a function of pressure gradient C, radial coordinate r, tube semi-diameter R, critical radius r_0 (or yield stress τ_0), Bingham fluid viscosity μ_a and u_s axial velocity slip at the boundary.

Important observations of the present model include the following:

i) If shear stress τ_{rz} at a distance is not higher than a finite yield stress, blood will not flow and

ii) If shear stress is not lower than its yield value, blood flow will be possible.

iii) The present model includes Poiseuille flow models with velocity slip and zero ship at vessel wall and steady one-dimensional Bingham plastic fluid model with zero wall slip, as its special cases.

iv) Flow variables indicate distinct behaviour for vanishing and non-vanishing yield stress.

v) Flow variables indicate distinct behaviour for different Hartmann number at different yield stress.

vi) Velocity profiles indicate a parabolic profile in all five arteries and for slip and no-slip cases with the usual maximum magnitude at tube axis and a minimum velocity at the boundary in case of vanishing yield stress.

vii) Velocity profiles indicate distinct bluntness near the axis for all five locations i.e., aorta, femoral, carotid, coronary and arteriole in CVS for non-vanishing yield stress. Therefore, non-Newtonian character of blood is clearly revealed in all five cases of CVS and this blunted or non-parabolic profile is found prominent with an increasing tube radius or with an increasing yield stress.

viii) Flow in the central region shows a flat profile for non-zero yield stress. Core region increases with an increase in yield stress and with an increase in tube radius.

ix) Velocity profile increases when Hartmann number M increases in different fluid parameter viz., yield stress $\tau_0(\geq 0)$. The nature of velocity profile is also same in no slip.

Theoretical analysis developed in this model, is based on the certain assumption which may result some physiological implications as follows:

i) Blood flow is assumed to be steady which is obviously true for very thin arteries in CVS wherein the pulsatile effects are small and so, these effects can be ignored.
ii) Further, velocity variation in axial direction is negligible as compared to its variation in radial direction, may lead to the implication that the length of the artery is too large as compared to the radius.

In this study, the effect of magnetic field on poiseuille flow of Bingham plastic fluid model for blood with velocity slip and no slip is examined. The application of Magneto dynamics in physiological flow problem is of growing interest. The flow of blood can be controlled by applying magnetic field. Mathematical modeling for poiseuille flow of blood (-a Bingham plastic fluid model) with an axial velocity slip along an artery wall in presence of magnetic field, is considered. It is observed that when Hartmann number increases the fluid velocity is greatly affected. The present model includes the poiseuille flow models of slip and no slip at artery wall and one- layered Bingham plastic fluid model with zero-slip, as its special cases. Applications of this theoretical modeling to cardiovascular diseases and the role of slip in the better functioning of the diseased or occluded arteries are included in brief.

Table 2.1. Data for five different locations in Cardiovascular System (CVS)

S. No.	Name of an artery	Radium (R*) x 10^{-2}m	Pressure gradient (C*) x 10 kg. $m^{-2}.s^{-2}$			r^0/R**
				$\tau_0 = 0.00$	$\tau_0 = 0.04$	$\tau_0 = 0.10$
01	Aorta	1.00	1.46	0.0000	0.0548	0.1370
02	Femoral	0.50	6.40	0.0000	0.0250	0.0625
03	Carotid	0.40	10.00	0.0000	0.0200	0.0500
04	Coronary	0.15	139.74	0.0000	0.0038	0.0095
05	Arteriole	0.008	400.00	0.0000	0.0250	0.0625

Sud and Sekhon (1985)
Present modeling (2001)

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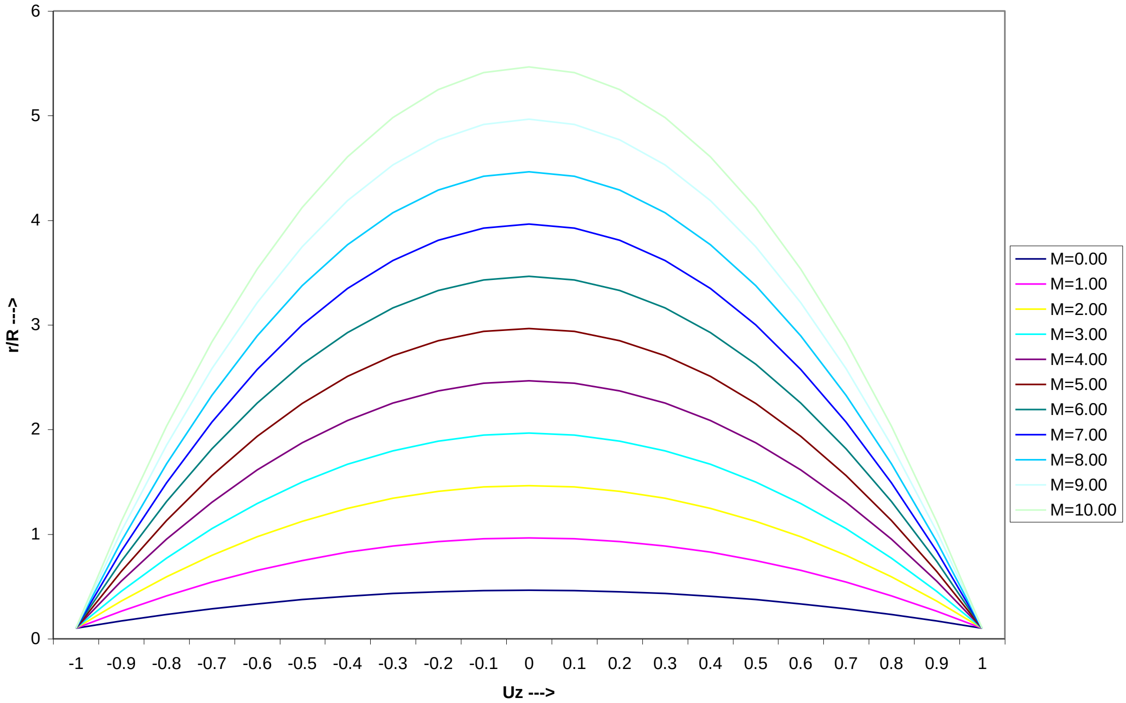
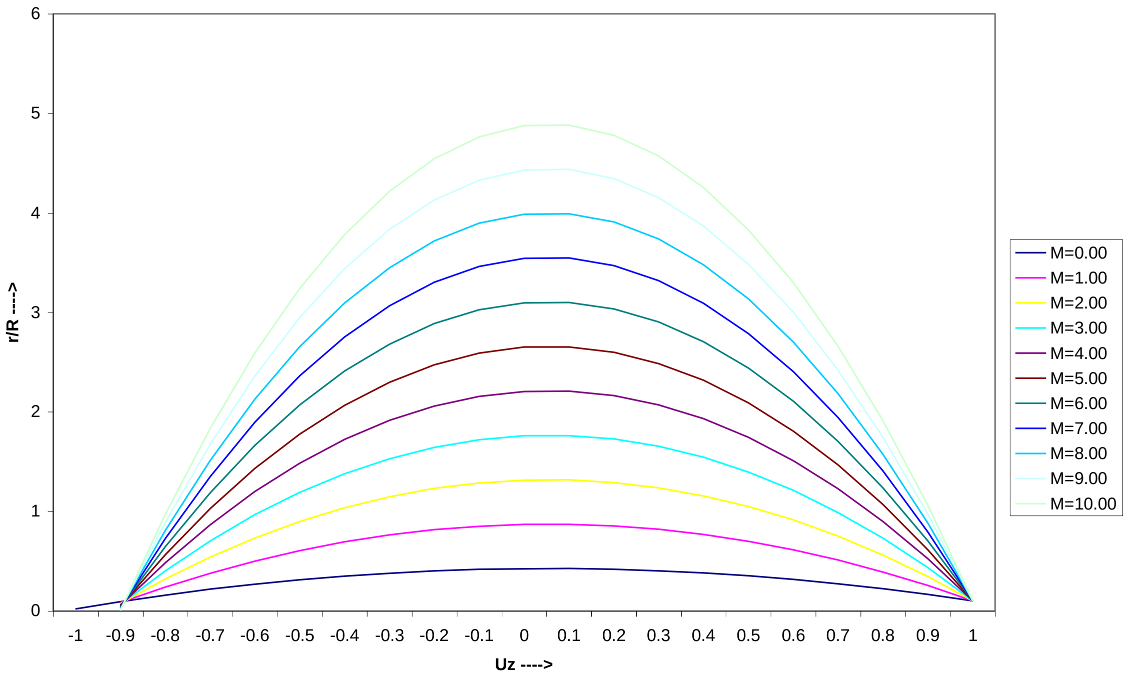


Fig. 1. Variation of velocity profiles Uz with Hartmann number M in Aorta when T0=0



Variation of velocity profiles Uz with Hartmann number in Aorta when T0=0.04

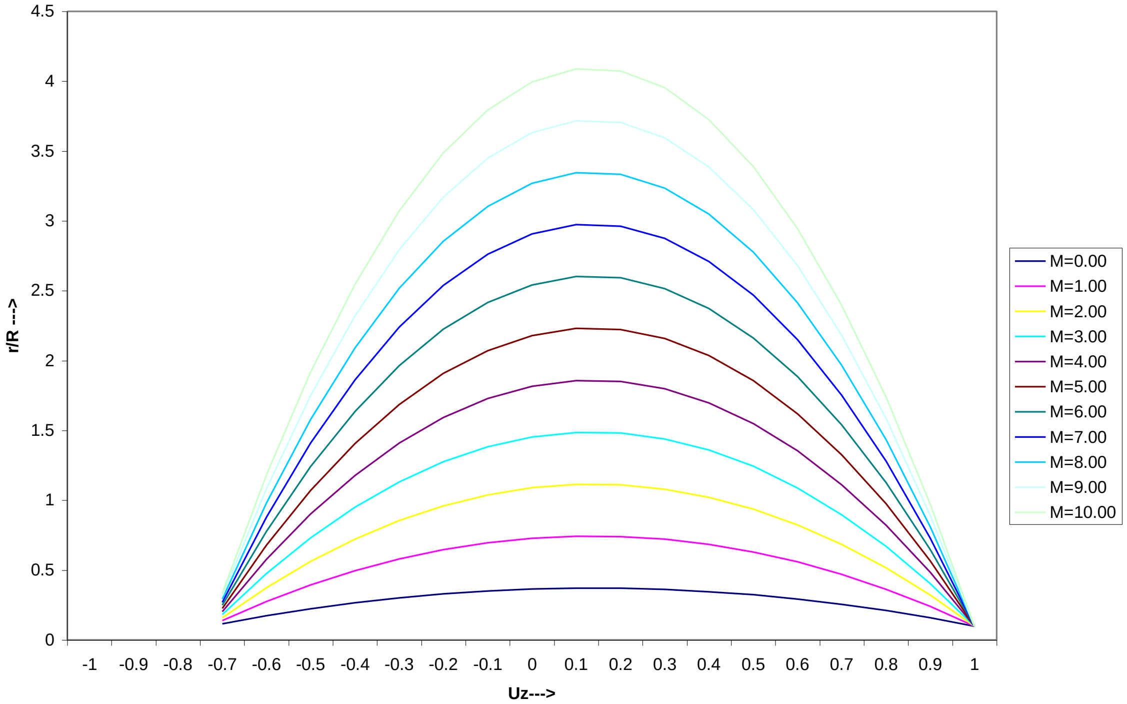


Fig. 3. Variation of velocity profiles with Hartmann number M when T0=0.10

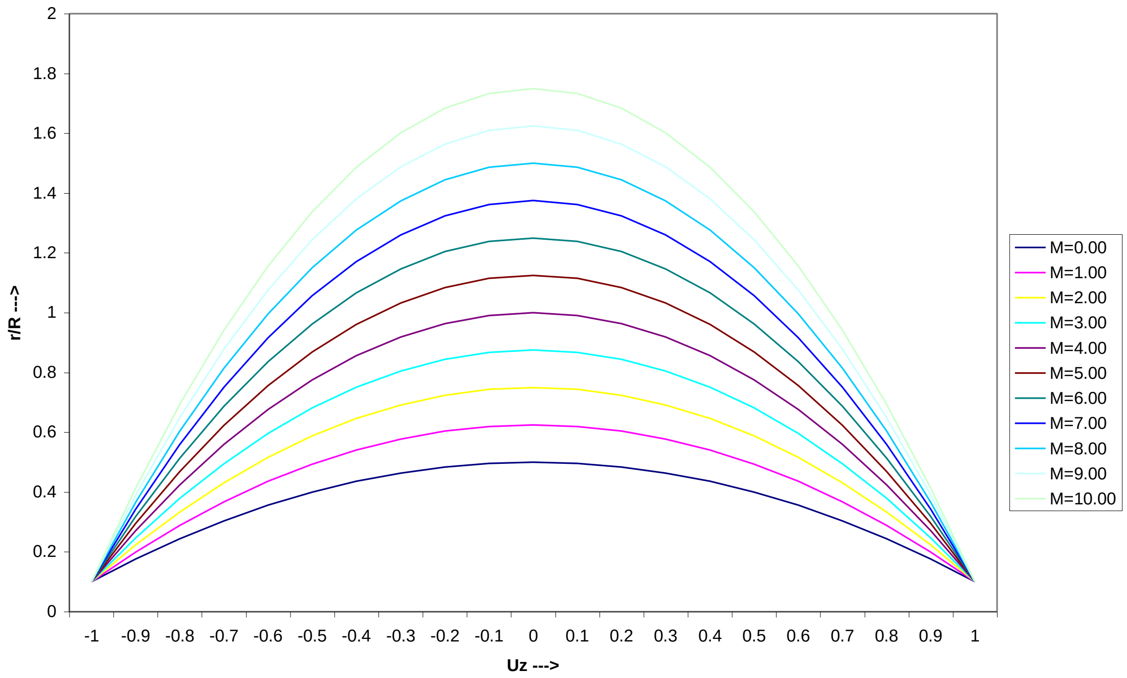


Fig. 4. Varation of velocity profiles Uz with Hartmann number M in femoral when T0=0

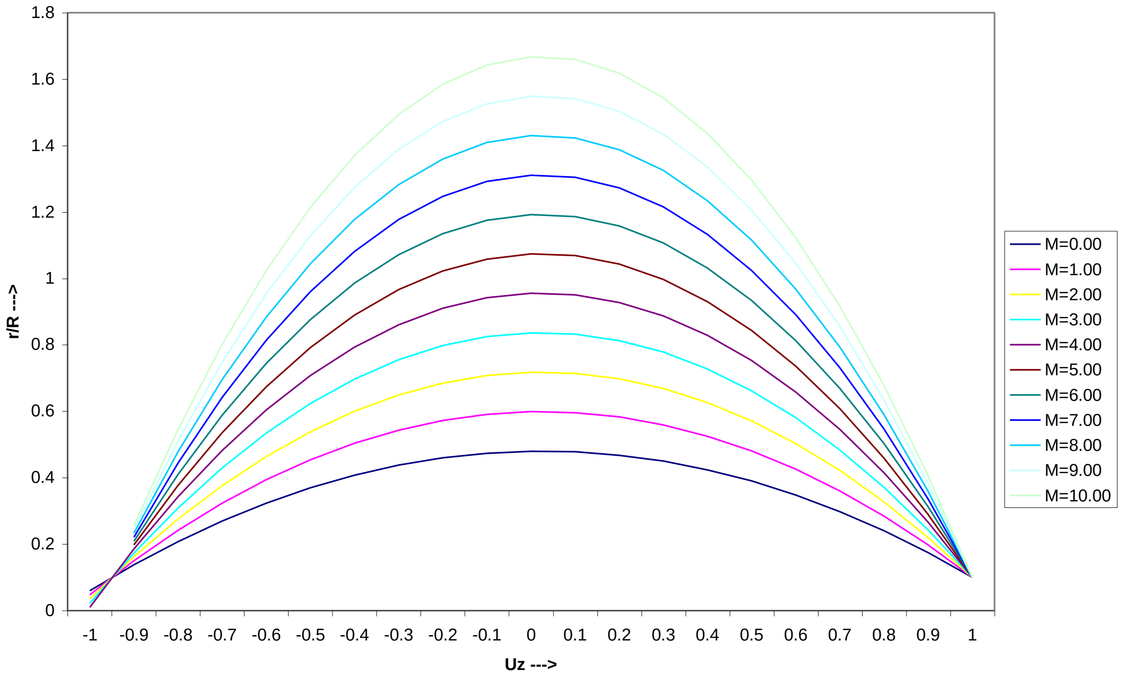


Fig. 5. Variation of Velocity profiles Uz with Hartmann number in Femoral when T0=0.04

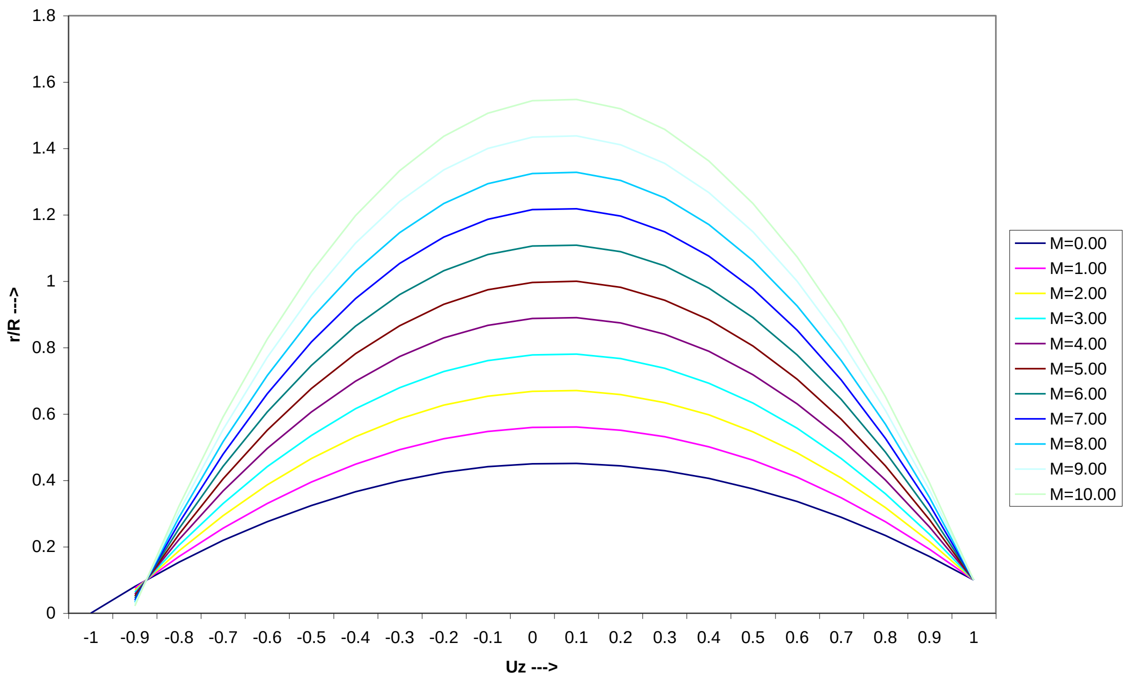


Fig. 6. Variation of velocity profiles Uz with Hartmann number M at femoral when T0=0.10

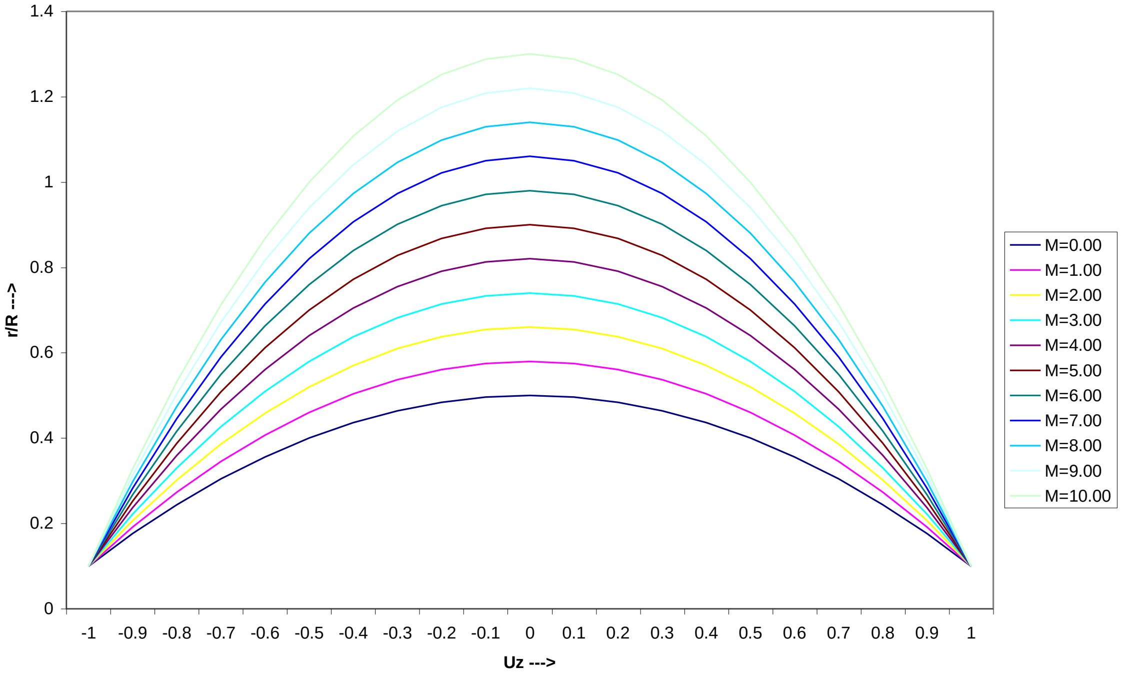


Fig. 7. Variation of velocity profiles Uz with Hartmann number M at Carotid when T0=0

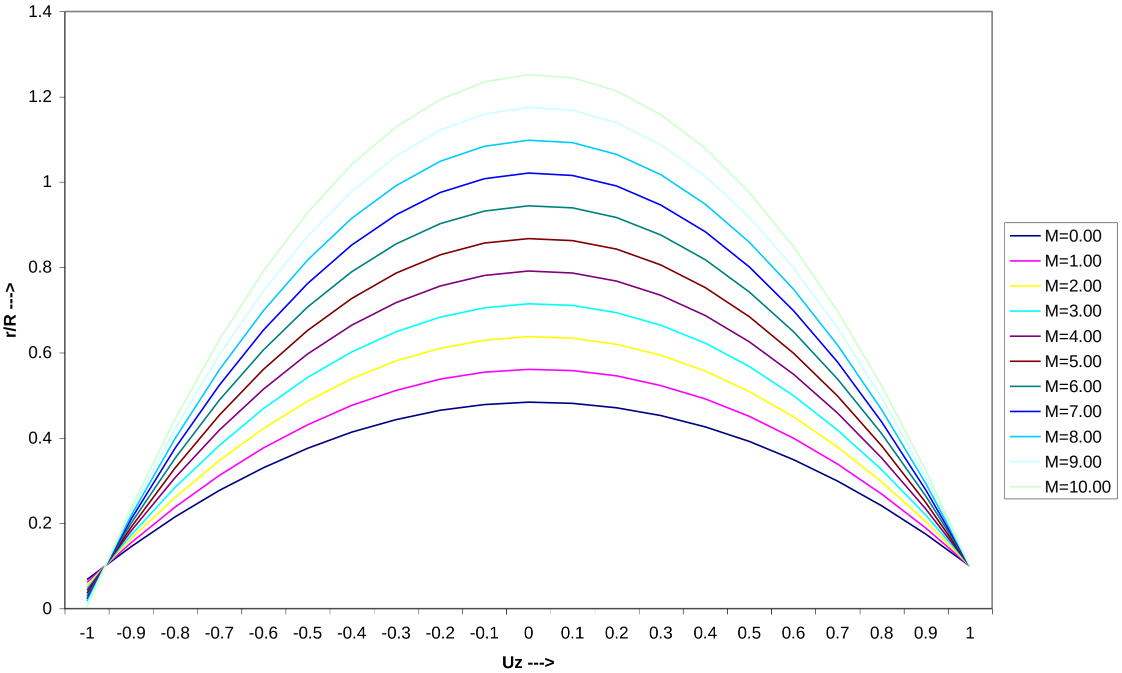


Fig. 8. Variation of Velocity profiles Uz with Hartmann number M at Carotid when T0=0.04

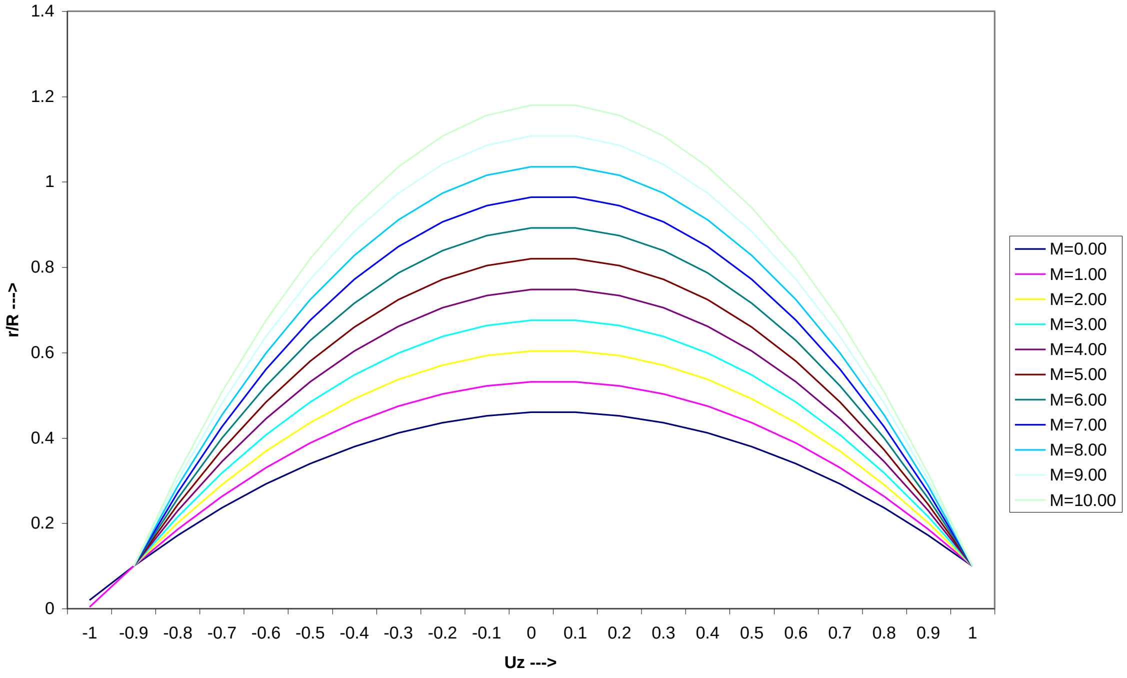


Fig. 9. Variation of velocity profiles Uz at Hartmann number M at Caritod when T0=0.10

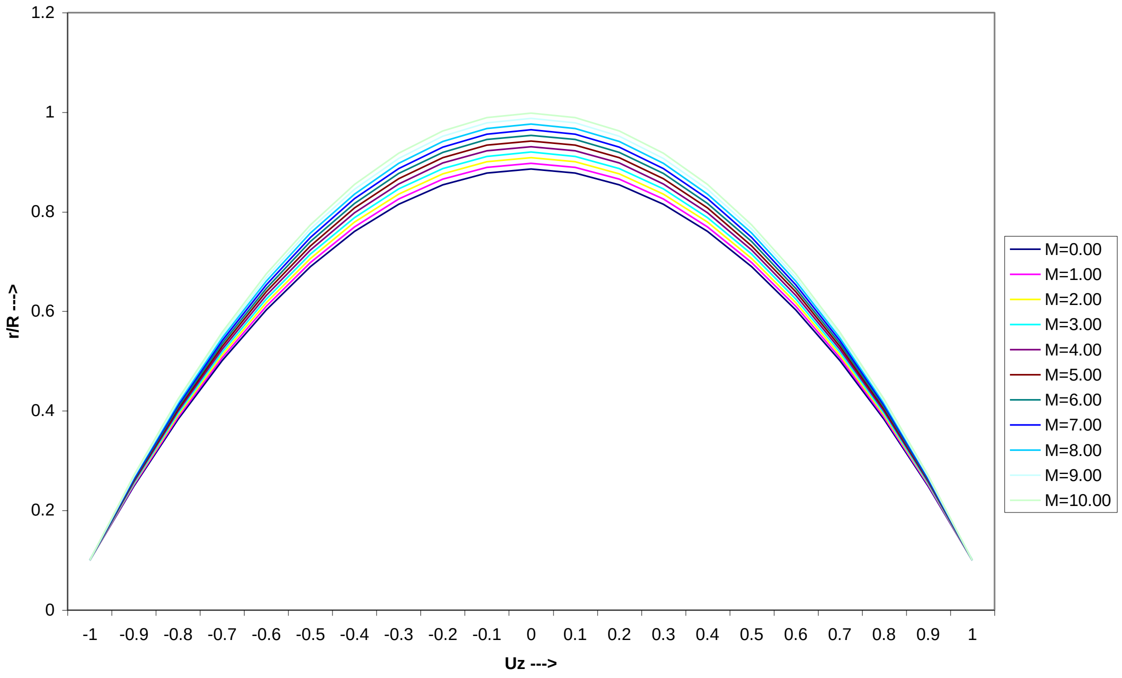


Fig. 10. Variation of velocity profiles Uz at Hartmann number M at Coronary when T0=0

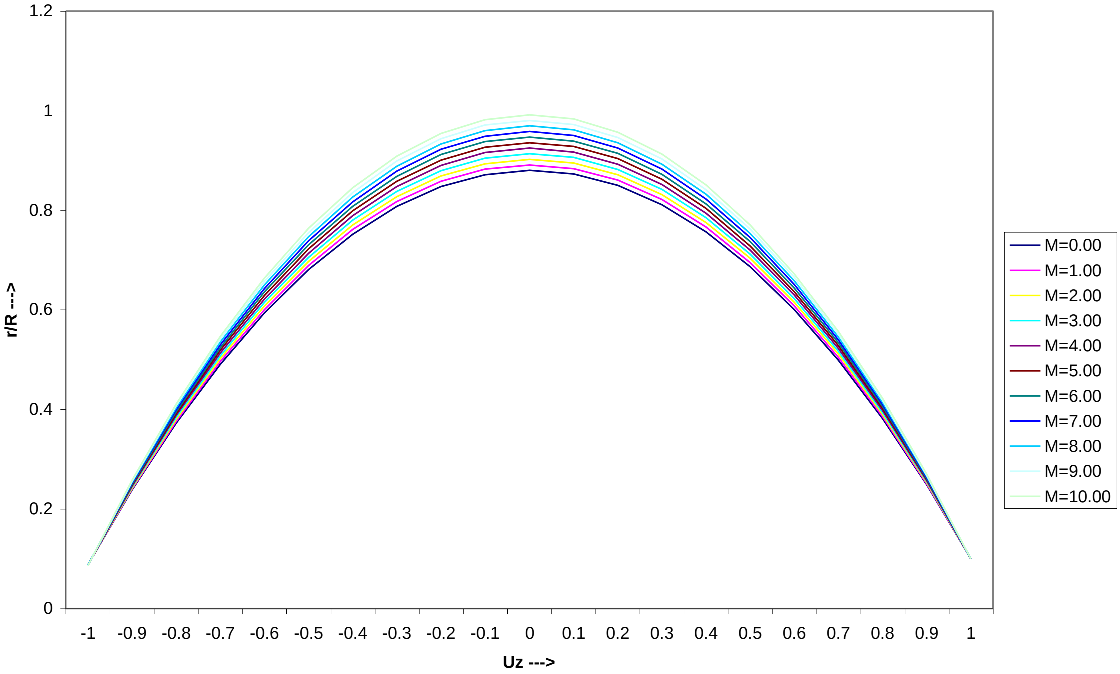


Fig. 11. Variation of velocity profiles Uz with Hartmann number M at Caritod when T0=0.04

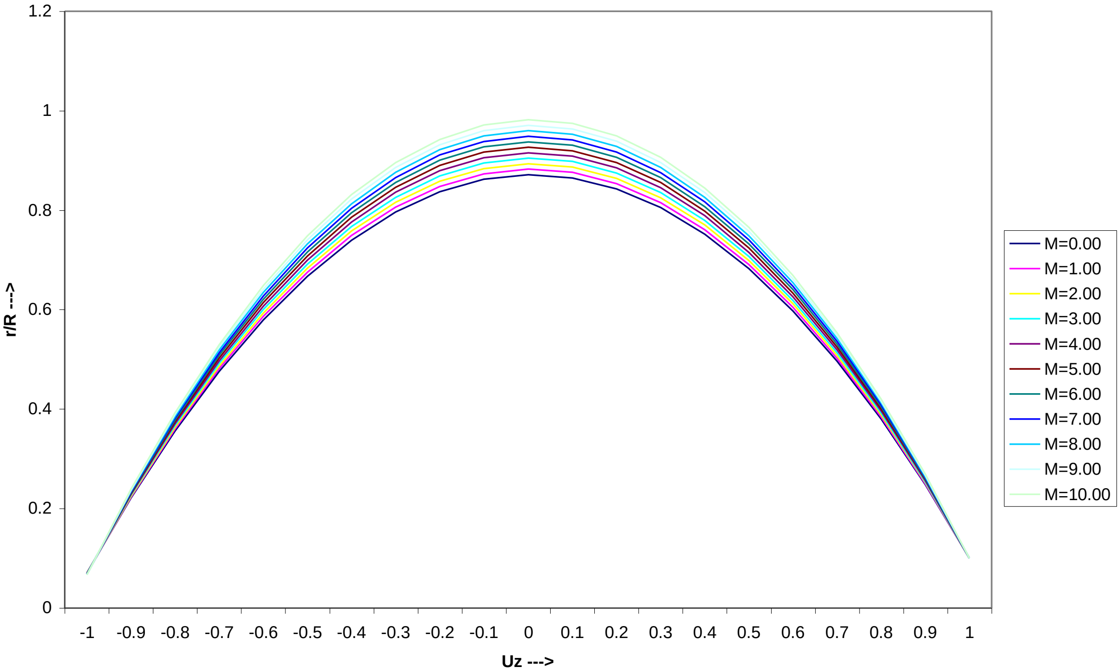


Fig. 12. Variation of velocity profiles Uz with Hartmann number M at Caritod when T0=0.10

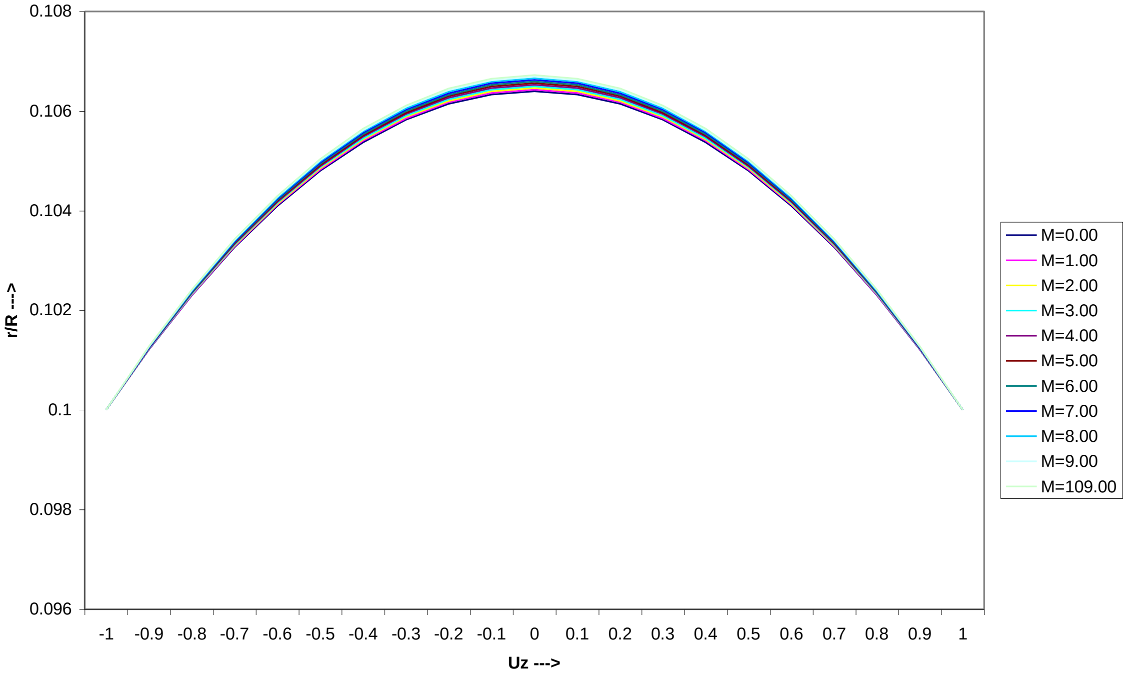


Fig. 13. Variation of velocity profiles Uz with Hartmann number M at Arteriole when T0=0

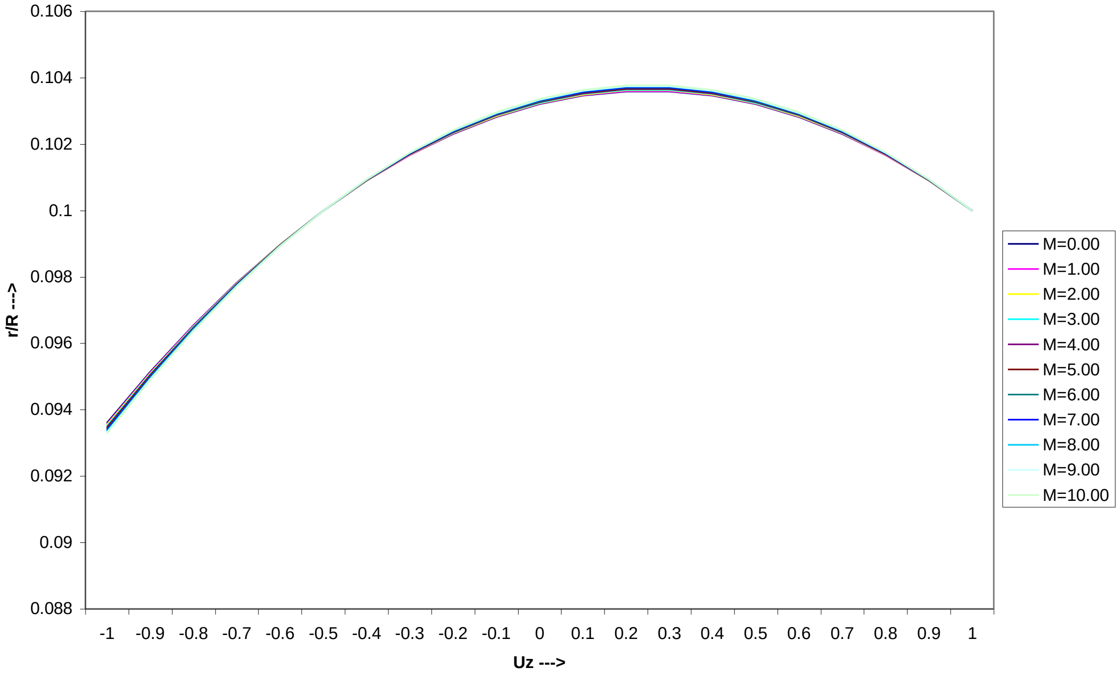


Fig. 14. Variation of velocity profiles Uz with Hartmann number M at Arteriole when T0=0.04

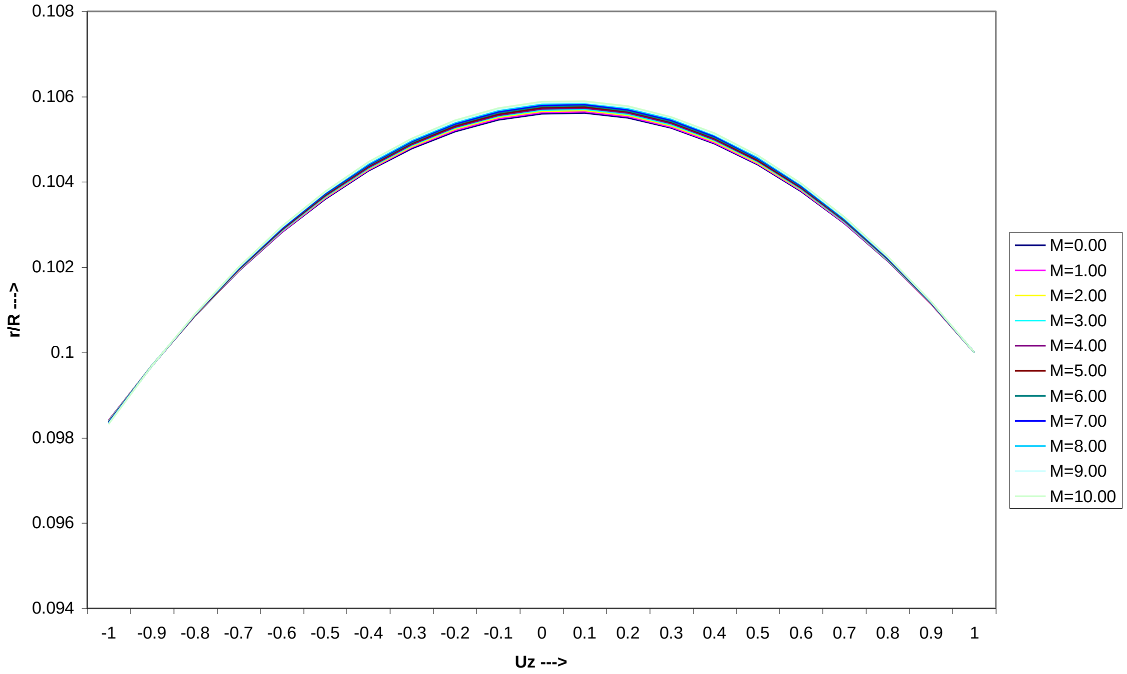


Fig. 15. Variation of velocity profiles Uz with Hartmann number M at Arteriole when T0=0.10