

Double-Diffusive Instability of Couple-Stress Binary Rivlin-Ericksen Visco-Elastic Fluid Mixture in Porous Medium

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Abstract: The paper critically examines, within the framework of linear stability analysis, the double diffusive instability of couple-stress binary Rivlin-Ericksen visco-elastic fluid mixture in porous medium. In the present paper, by a number of theorems providing conditions for stability or instability, bounds on the growth rate of unstable modes and the characterization of modes providing clear cut effects of viscosity and medium permeability.

Keywords: Couple stress, Binary fluid mixture, Vertical temperature and concentration gradient, Stability conditions, Rivlin-Ericksen fluids.

I. INTRODUCTION

The stability of couple stress binary fluid mixture having vertical temperature and concentration gradients was first investigated by Elshehawey and Abou-Tair¹. It is well known that in an isothermal, viscous and heat-conducting fluid, transverse thermal waves (i.e., pure thermal diffusion waves) are strongly damped, and this was critically examined by Carslaw and Jaeger². Luikov and Berkovsky³ showed for the first time that if an unbounded fluid, at rest in undisturbed state, is heated from below, the above two pure diffusion waves propagating in the horizontal direction become coupled and give rise to two modes of modified waves. Though the transport of wave energy is due to viscosity and thermal conductivity of the fluid, one of these two modes can be damped. These weakly damped waves are known as thermo-convective waves. The interaction of thermo-convective and sound waves and the amplitude of the various wave modes in a compressible fluid were investigated by Keck and Schneider⁴. Keck⁵ further extended this problem to a thermally radiating fluid and obtained interesting interaction between thermo-convective and radiation-induced waves. The theoretical study of the propagation of thermo-convective waves was made by Berkovsky and Sinitsyn^{6, 7} and the experimental study was conducted by Barkov et al.⁸. Bhattacharya and Gupta⁹ investigated the propagation of thermo-convective waves in an unbounded binary fluid mixture, which is at rest in the undisturbed state, having vertical temperature and concentration gradients and obtained conditions under which undamped/weakly damped thermo-convective waves propagate. Takishama¹⁰ investigated the problem of Bhattacharya and Gupta and his finding are not in agreement with the conditions obtained by Gupta and Bhattacharya.

The study of thermo-convective waves was extended to the fluids with peculiar properties (i.e., visco-elastic, ferromagnetic or electrically conducting fluids) by a number of research workers, mainly, Luikov and Berkovsky¹¹, Gupta and Gupta¹², Berkovsky et al.¹³, Takishama^{14,15}, Hamabata and Namikawa¹⁶ and Hamabata¹⁷. It is recognized that the couple stresses appear in noticeable magnitudes in polymer solutions (liquids with large molecules). The field equations for the couple stress vector were discussed by Cosserat and Cosserat¹⁸ and a modern derivation was given by Truesdell and Toupin¹⁹. Dahler and Seriven²⁰ also discussed the constitutive equations for the couple stresses in fluids. Boundary value problems arising in fluids with couple stresses were discussed by Condiff and Dahler²¹. Stokes²² provided an excellent discussion of force and couple stresses and proposed a set of constitutive equations. Elshehawey and Abou Tair have made an important contribution to the study of couple stress binary fluid mixture having vertical temperature and concentration gradients. The objective was to provide a simple generalization of the classical theory so as to allow the polar effects. Their analysis is based upon the constitutive equations for force and couple stresses proposed by Stokes. Restricting to two-dimensional disturbances and using the Hurwitz's conditions, the only result obtained by them is a sufficient condition for the stability of the system.

Rachna and Agrawal²³ have made an attempt to reinvestigate the problem of Elshehawey and Abou Tair. The investigation is necessary in view of the fact that neither the sufficient conditions obtained by them provide clear cut effect of various parameters occurring in the problem nor their analysis provides any characterization of modes, condition for the existence of unstable modes, bounds on the growth rate of unstable mode, etc. Rachna

and Agrawal obtained the sufficient conditions for stability and instability, necessary condition for the existence of non-oscillatory modes, bounds on the growth rate of these unstable modes under certain conditions.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. The Rivlin-Ericksen²⁴ is one such fluid. Many researchers have paid their attention towards the study of Rivlin-Ericksen fluid. Johari²⁵ has discussed the visco-elastic Rivlin-Ericksen incompressible fluid under time dependent pressure gradient. Sisodia and Gupta²⁶ and Srivastava and Singh²⁷ have studied the unsteady flow of a dusty elastico-viscous Rivlin-Ericksen fluid through channel of different cross-section in the presence of time dependent pressure gradient. Sharma and Kumar²⁸ have studied the thermal instability of a layer of Rivlin-Ericksen elastico-viscous fluid acted upon by a uniform rotation and found that rotation has a stabilizing effect and introduces oscillatory modes in the system. Sharma, Chand and Sunil²⁹ have studied the thermosolutal instability of Rivlin-Ericksen rotating fluid in porous medium. Nidhi, Jaimala, Agrawal³⁰ have investigated the shear flow instability of an incompressible visco-elastic fluid in a porous medium in the presence of a weak magnetic field.

In view of the fact that the study of Rivlin-Ericksen visco-elastic fluids in a porous medium may find applications in geophysics and chemical technology, a number of researchers have contributed in this direction. Thermosolutal instability of couple stress binary Rivlin-Ericksen visco-elastic fluid mixture in porous medium seems, to the best of our knowledge, uninvestigated so far.

In the present paper, therefore, we have made an attempt to examine the effect of Rivlin-Ericksen visco-elastic fluid on the stability of couple stress binary fluid mixture having vertical temperature and concentration gradients. Hence, it can be looked upon as an extension of the work on the stability of couple stress binary fluid mixture having vertical temperature and concentration gradients, discussed by Rachna and Agrawal.

II. EQUATIONS OF MOTION

Consider the infinite space $-\infty < x, y, z < \infty$ occupied by a quasi-incompressible, Stoke's couple stress and heat and electrically conducting fluid consisting of two components, where a coordinate system (x, y, z) has been chosen, with z -axis vertically upward. Following Elsbehawey and Abou Tair, the governing equations using Rivlin-Ericksen visco-elastic model are

$$\frac{\rho}{\phi} \left[\frac{\partial \mathbf{V}}{\partial t} + \frac{1}{\phi} (\mathbf{V} \cdot \nabla) \mathbf{V} \right] = \rho \mathbf{g} - \nabla p - \frac{1}{\kappa_1} \left(\mu + \mu' \frac{\partial}{\partial t} \right) \mathbf{V} - \eta \nabla^4 \mathbf{V}, \quad (1)$$

$$\nabla \cdot \mathbf{v} = 0, \quad (2)$$

$$\phi \frac{\partial T}{\partial t} + (\mathbf{V} \cdot \nabla) T = \kappa \nabla^2 T, \quad (3)$$

$$\phi \frac{\partial C}{\partial t} + (\mathbf{V} \cdot \nabla) C = \kappa' \nabla^2 C \quad (4)$$

$$\text{and } \rho = \rho_0 \{1 - \alpha(T - T_0) - \alpha'(C - C_0)\}, \quad (5)$$

where $\mathbf{V}, \rho, p, \mu, \eta, T, C, \kappa, \kappa', \alpha, \alpha', \mu', \kappa_1$ and ϕ are respectively the fluid velocity, density, pressure, dynamic viscosity coefficient, material constant, temperature, solute concentration, thermal diffusivity, solute diffusivity, thermal expansion coefficients, solute expansion coefficient, viscoelasticity, medium permeability and medium porosity.

Here, it is assumed that, when a fluid passes through a porous medium, the gross effect is represented by

$$-\left(\frac{\mu}{\kappa_1} \right) \mathbf{V}.$$

Darcy's law. As a result, the usual viscous term is replaced by the resistance term

III. BASIC STATE AND THE PERTURBATION EQUATIONS

The time independent solution of equations (1) to (5) (known as the basic state), whose stability we wish to examine is that of an incompressible, Rivlin-Ericksen visco-elastic fluid of varying density and variable viscosity, in a homogeneous and isotropic porous medium. The system is acted upon by a temperature field T ,

concentration field C and gravity field $\vec{g} = (0, 0, -g)$. The character of equilibrium is examined by supposing that the system is slightly disturbed and then by following its further evolution.

We suppose that the Stoke's couple stress fluid is at rest and the constant vertical temperature and concentration gradients are maintained in the fluid. The basic state is, therefore, described as

$$\mathbf{V} = (0, 0, 0), T = T_0 - \alpha z \quad \text{and} \quad C = C_0 - \alpha' z, \tag{6}$$

where α and α' may be either positive or negative and this basic state is consistent with equations (1) to (5) provided that

$$\left. \begin{aligned} \frac{\partial p}{\partial z} - \rho g &= 0, \\ \rho &= \rho_0(1 + (\alpha\beta + \alpha'\beta')z) \\ \text{and} \quad p &= p_0 - \rho_0 g \left[z + \frac{1}{2}(\alpha\beta + \alpha'\beta')z^2 \right] \end{aligned} \right\} \tag{7}$$

Now, let the velocity, temperature, concentration, density and pressure in the perturbed state be given by

$$\left. \begin{aligned} \mathbf{V} &= (u, v, w), \\ T &= T_0(z) + \theta, \\ C &= C_0(z) + \Gamma, \\ \rho &= \rho_0(z) + \rho \\ \text{and } p &= p_0(z) + p \end{aligned} \right\} \tag{8}$$

Here, suffix zero refers to the values at the reference level $z = 0$. Now, using equations (8) and (1) to (5), the linearized perturbation equations are

$$\frac{\rho_0}{\phi} \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} - \eta \nabla^4 u - \frac{\mu}{\kappa_1} u - \frac{\mu'}{\kappa_1} \frac{\partial u}{\partial t}, \tag{9}$$

$$\frac{\rho_0}{\phi} \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} - \eta \nabla^4 v - \frac{\mu}{\kappa_1} v - \frac{\mu'}{\kappa_1} \frac{\partial v}{\partial t}, \tag{10}$$

$$\frac{\rho_0}{\phi} \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} - \rho g - \eta \nabla^4 w - \frac{\mu}{\kappa_1} w - \frac{\mu'}{\kappa_1} \frac{\partial w}{\partial t}, \tag{11}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{12}$$

$$\left(\phi \frac{\partial}{\partial t} - \kappa \nabla^2 \right) \theta = \beta w, \tag{13}$$

$$\left(\phi \frac{\partial}{\partial t} - \kappa' \nabla^2 \right) \Gamma = \beta' w, \tag{14}$$

$$\text{and} \quad \rho = -\rho_0(\alpha\theta + \alpha'\Gamma). \tag{15}$$

Analyzing the perturbations into normal modes, we seek solutions, whose dependence on x, y and t is given by $\exp[i(ax + by + cz) + nt]$, (16)

where a, b and c are real wave number and n , a time constant, is complex, in general.

On eliminating various physical quantities from the resulting equations in favour of w and Γ , we have

$$\left[\frac{\rho_0}{\phi} n + \eta l^4 + \frac{\mu}{\kappa_1} + \frac{\mu'}{\kappa_1} n \right] l^2 (\phi n + \kappa l^2) (\phi n + \kappa l^2) = g \rho_0 m^2 [\alpha \beta (\phi n + \kappa l^2) + \alpha' \beta' (\phi n + \kappa l^2)] \tag{17}$$

where $l^2 = a^2 + b^2 + c^2$ and $m^2 = a^2 + b^2$.

IV. TWO DIMENSIONAL DISTURBANCES

Hence, we shall consider two dimensional disturbances in x, y plane so that we shall have $c = 0$. Then $l = m$.

Therefore, equation (17) becomes

$$\left[\frac{\rho_0}{\phi} n + \eta l^4 + \frac{\mu}{\kappa_1} + \frac{\mu'}{\kappa_1} n \right] [(\phi n + \kappa' l^2)(\phi n + \kappa l^2)] = g\rho_0\alpha\beta(\phi n + \kappa' l^2) + g\alpha'\beta'\rho_0(\phi n + \kappa l^2) \tag{18}$$

On dividing (18) by \square_0 , we have

$$\left[\frac{n}{\phi} + v_2 l^4 + \frac{v_1}{\kappa_1} + \frac{v'}{\kappa_1} n \right] [(\phi n + \kappa' l^2)(\phi n + \kappa l^2)] = g\alpha\beta(\phi n + \kappa' l^2) + g\alpha'\beta'(\phi n + \kappa l^2), \tag{19}$$

where $v_1 = \frac{\mu}{\rho_0}$, $v_2 = \frac{\eta}{\rho_0}$ and $v' = \frac{\mu'}{\rho_0}$.

Expressing the co-ordinates x and y in new unit d-the length and letting,

$$a = ld, \sigma = nd^2 / v_1, P_r = \frac{v_1}{\kappa}, S = \frac{v_1}{\kappa}, P_m = \frac{v_2}{\kappa^2}, \kappa^2 = \kappa l^2, F = v' / \kappa', P_l = \kappa_1 / d^2,$$

Equation (19) becomes

$$\left(\frac{\sigma v_1}{\phi d^2} + v_2 \frac{a^4}{d^4} + \frac{v_1}{\kappa_1} + \frac{v'}{\kappa_1} \frac{\sigma v_1}{d^2} \right) \left(\phi \frac{\sigma v_1}{d^2} + \kappa' \frac{a^2}{d^2} \right) \left(\phi \frac{\sigma v_1}{d^2} + \kappa \frac{a^2}{d^2} \right) = g\alpha\beta \left(\phi \frac{\sigma v_1}{d^2} + \kappa' \frac{a^2}{d^2} \right) + g\alpha'\beta' \left(\phi \frac{\sigma v_1}{d^2} + \kappa \frac{a^2}{d^2} \right)$$

After simplification, we get

$$\begin{aligned} & (\phi + \phi^2 F)\sigma^3 + \sigma^2 \left\{ \phi^2 \frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l} \phi^2 + \frac{a^2}{S} + \phi_l F \frac{a^2}{S} + \frac{a^2}{P_r} + \phi F \frac{a^2}{P_r} \right\} \\ & \sigma \left\{ \left(\frac{1}{\phi} + F \right) \frac{a^4}{P_r S} + \phi \left(\frac{a^2}{S} + \frac{a^2}{P_r} \right) \left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l} \right) - g\alpha\beta \frac{d^4}{v_1^2} \phi - g\alpha'\beta' \frac{d^4}{v_1^2} \phi \right\} \\ & \left\{ \left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l} \right) \frac{a^4}{P_r S} - \frac{g\alpha\beta d^4 a^2}{v_1^2 S} - \frac{g\alpha'\beta' d^4 a^2}{v_1^2 P_r} \right\} = 0, \tag{20} \end{aligned}$$

where $k^2 = \square(a^2 + b^2)$.

Equation (20) can also be written as

$$\square^3 + B\square^2 + C\square + D = 0, \tag{21}$$

where $B = \frac{\phi^2 \frac{P_m}{P_r} a^2 k^2 + \frac{\phi^2}{P_l} + \frac{a^2}{S} + \phi F \frac{a^2}{S} + \frac{a^2}{P_r} + \phi F \frac{a^2}{P_r}}{\phi(1 + \phi F)} = \frac{\phi \left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l} \right)}{(1 + \phi F)} + \frac{1}{\phi} \left(\frac{a^2}{S} + \frac{a^2}{P_r} \right)$

$$C = \frac{\left[\left(\frac{1}{\phi} + F \right) \frac{a^4}{P_r S} + \phi \left(\frac{a^2}{S} + \frac{a^2}{P_r} \right) \left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l} \right) - \frac{g\alpha\beta d^4}{v_1^2} \phi - \frac{g\alpha'\beta' d^4}{v_1^2} \phi \right]}{\phi(1 + \phi F)}$$

and $D = \frac{\left[\left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l} \right) \frac{a^4}{P_r S} - \frac{g\alpha\beta d^4 a^2}{v_1^2 S} - \frac{g\alpha'\beta' d^4 a^2}{v_1^2 P_r} \right]}{\phi(1 + \phi F)}$

It is important to note that $A = \square \square g$, $A' = \square' \square' g$ can be both positive or negative depending upon whether the temperature/concentration decreases or increases in the vertically upward direction.

V. RESULTS AND DISCUSSION

(Case A) – D < 0

Note that D is negative under the condition

$$\left(\frac{A}{S} + \frac{A'}{P_r}\right) \frac{d^4 a^2}{v_1^2} > \left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l}\right) \frac{a^4}{P_r S}.$$

Theorem-1. System is unstable if D < 0.

Proof : Let $\square_1, \square_2, \square_3$ be the roots of the equation (21). Then the product of the roots is positive if D < 0. This ensures the existence of at least one positive root which implies the instability of the system. The unstable regions in A-A' plane is shown in figure 1.

(i) The unstable region is obtained beyond the line

$$\frac{A}{e} + \frac{A'}{f} = 1,$$

where $e = \frac{v_1^2 \left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l}\right) a^2}{P_r d^4}$ and $f = \frac{v_1^2 \left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l}\right) a^2}{S d^4}$.

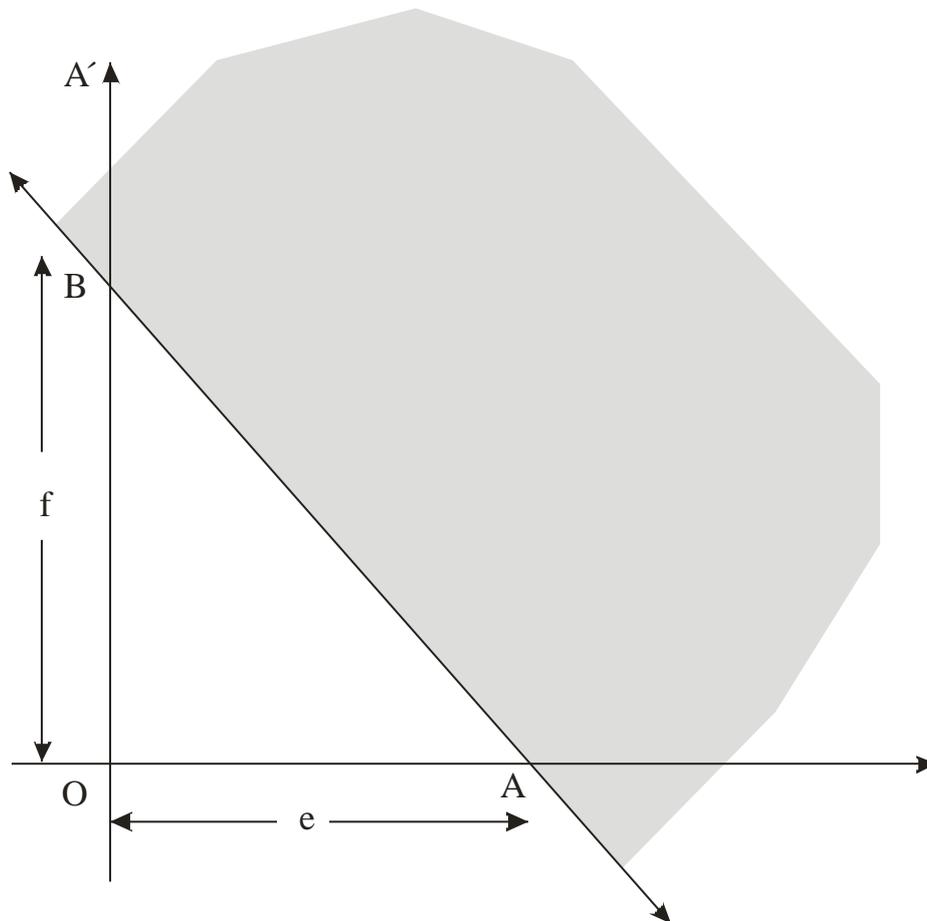


Fig. 1

Effect of Viscosity and Medium Permeability

The area of the triangle (stable region) $OAB = \frac{1}{2} \times e \times f$

$$= \frac{1}{2} \cdot \frac{v_1^4 \left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l} \right)^2 d^4}{P_r \cdot S \cdot d^8} = \frac{P_r^4 \kappa^4 \left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l} \right)^2 d^4}{2 P_r \cdot S \cdot d^8}$$

In the absence of Rivlin-Ericksen fluid, discussed by Rachna and Agrawal, the area of the triangle (Stable region)

$$= \frac{k^8 P_r^3 \left(1 + \frac{P_m}{P_r} k^2 \right)}{2S}$$

Comparing the above two regions, it is found that, if

$$P_r > P_m \cdot P_l k^4 \left[\frac{d^4}{a^2} - \frac{a^2}{k^2} \right] + \frac{d^4}{a^2},$$

then the unstable region obtained by us is less than the unstable region obtained by Rachna and Agrawal, showing thereby the stabilizing effect of viscosity. On the other hand, if

$$P_r < P_m \cdot P_l k^4 \left[\frac{d^4}{a^2} - \frac{a^2}{k^2} \right] + \frac{d^4}{a^2},$$

then the unstable region obtained by us is greater than the unstable region obtained by Rachna and Agrawal, showing thereby the destabilizing effect of viscosity. Hence, viscosity has a dual character.

Similar discussion follows for the dual character of medium permeability.

Theorem-2. The unstable modes which exist under the condition $D < 0$ are non-oscillatory. **Proof:** The equation (21) can be written as

$$\sigma^2 + B\sigma + C + \frac{D}{\sigma} = 0. \tag{22}$$

Taking imaginary part of (22), we have

$$\sigma_i \left(2\sigma_r + B - \frac{D}{|\sigma|^2} \right) = 0 \tag{23}$$

Now, B is always positive. Also $D < 0$ and since the modes are unstable, therefore, $\sigma_r > 0$. Therefore, for the consistency of equation (23), we must have $\sigma_i = 0$, which implies that modes are non-oscillatory.

Hence, the theorems 1 and 2, when combined, yield that if $D < 0$, then the modes are unstable and non-oscillatory.

Theorem-3. Estimates on σ_r for the existence of non-oscillatory unstable modes, when $C > 0$ are given by

$$\sigma_r < \text{Min} \left\{ \frac{|D|}{C}, \sqrt{\frac{|D|}{B}}, |D|^{1/3} \right\}$$

provided both the temperature and concentration gradients increase in the vertically upward direction.

Proof: For non-oscillatory unstable modes, we must have

$$\sigma_r > 0 \text{ and } \sigma_i = 0.$$

Now, when $D < 0$, equation (21) becomes

$$\sigma_r^3 + B\sigma_r^2 + C\sigma_r - |D| = 0. \tag{24}$$

For the consistency of the equation (24), we must necessarily have

$$C\sigma_r - |D| < 0$$

which implies $\sigma_r < \frac{|D|}{C}$. (25)

Since, $B > 0$, and therefore, for the consistency of equation (24), we must necessarily have

$$B\sigma_r^2 - |D| < 0$$

which gives $\sigma_r < \sqrt{\frac{|D|}{B}}$. (26)

Also, $\sigma_r^3 - |D| < 0$

which implies $\sigma_r < |D|^{1/3}$. (27)

On combining inequalities (25), (26) and (27), we get the estimates on \square_r for the existence of non-oscillatory unstable modes under the condition $C > 0$ (temperature and concentration gradients increase in vertically upward direction) are given by

$$\sigma_r < \text{Min} \left\{ \frac{|D|}{C}, \sqrt{\frac{|D|}{B}}, |D|^{1/3} \right\}$$

Theorem-4. Estimates on \square_r for the existence of non-oscillatory unstable modes when $C < 0$ are given by

$$\sigma_r < \frac{|C| + \sqrt{|C|^2 + 4B|D|}}{2B}$$

provided either temperature or concentration gradient decrease in vertically upward direction. **Proof:** For $\square_r > 0$, $\square_i = 0, D < 0$ and $C < 0$, equation (21) can be written as

$$\sigma_r^3 + B\sigma_r^2 - |C|\sigma_r - |D| = 0. \tag{28}$$

Now, for the consistency of equation (28), we must necessarily have

$$B\sigma_r^2 - |C|\sigma_r - |D| < 0$$

which gives

$$\sigma_r < \frac{|C| + \sqrt{|C|^2 + 4B|D|}}{2B}$$

(Case B) – $D > 0$

Discussion for Non-Oscillatory Modes for $D > 0$

D is positive, either when both the temperature and concentration gradient increase in the vertically upward direction so that A and A' both are negative or if both do not increase, under the condition

$$P_r.A + S.A' < \left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l} \right) \frac{a^2 v_1^2}{d^4}.$$

Theorem-5. Non-oscillatory unstable modes cannot exist.

Proof : Let the non-oscillatory unstable modes exist when $D < 0$, then equation (21) becomes

$$\sigma_r^3 + B\sigma_r^2 + C\sigma_r - D = 0. \tag{29}$$

Here, B is positive definite.

Further since $B > 0, D > 0$ and for unstable modes, $\square_r > 0$, then for the consistency of equation (29), C must necessarily be negative. But, when both the temperature and concentration gradients increase in the vertically upward direction, then C will be positive. Hence, non-oscillatory unstable modes cannot exist when both A and A' are negative. In other words, we can say that if non-oscillatory modes exist under the condition stated in the theorem, then such modes are stable or if unstable modes exist, then they are oscillatory.

Theorem-6. The necessary condition for the existence of non-oscillatory unstable modes under the condition

$$P_r.A + S.A' < \left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l} \right) \frac{a^2 v_1^2}{d^4}$$

is that $C < 0$.

Proof : Let the non-oscillatory unstable modes exist under the condition stated in the theorem. Then for non-oscillatory unstable modes, we have

$$\square_r > 0 \text{ and } \square_i = 0.$$

and equation (29) reduces to

$$\sigma_r^3 + B\sigma_r^2 + C\sigma_r + D = 0. \tag{30}$$

Now, B is positive definite.

and when $P_r \cdot A + S \cdot A' < \left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l} \right) \frac{a^2 v^2}{d^4}$, then D is also positive definite. Also, $\square_r > 0$ for unstable modes. Then for the consistency of equation (30), C must be negative, i.e., $C < 0$.

Theorem-7. For $\left(\frac{A}{S} + \frac{A'}{S'} \right) \frac{d^4 a^2}{v_1^2} > \left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l} \right) \frac{a^4}{P_r S}$, there are three waves propagating for a given

wave number : two damped and one amplified.

Proof: Let the modes be non-oscillatory, then consider real part of equation (21)

$$\sigma_r^3 + B\sigma_r^2 + C\sigma_r + D = 0.$$

Let the roots of the equation (30) be $\sigma_{r_i}, i = 1, 2, 3$. Then using the theory of equations, we get

$$\sigma_{r_1} \cdot \sigma_{r_2} \cdot \sigma_{r_3} = -D$$

and $\sigma_{r_1} + \sigma_{r_2} + \sigma_{r_3} = -B.$

Clearly, when $\left(\frac{A}{S} + \frac{A'}{S'} \right) \frac{d^4 a^2}{v_1^2} > \left(\frac{P_m}{P_r} a^2 k^2 + \frac{1}{P_l} \right) \frac{a^4}{P_r S}$, then D is negative. Also, B is positive so that the

product of the roots is positive and sum of the roots is negative. Therefore, the possibility that all three non-oscillatory modes can be unstable is ruled out. It follows that two waves of propagation are damped and one is amplified for a given wave number.

(Case C) – D > 0 : Discussion for Oscillatory Modes

Theorem-8. The growth rate of arbitrary oscillatory unstable modes must lie inside the intersection of circular disc of radius $\sqrt{\frac{D}{B}}$ and the curve $\sigma_r^3 + \sigma_r \sigma_i^2 = \frac{D}{2}$.

Proof: Obviously B is positive definite. Then equation (21) becomes

$$\sigma^2 + B\sigma + C + \frac{D}{\sigma} = 0. \tag{31}$$

Taking imaginary part of equation (2.4.13), we have

$$\sigma_i \left(2\sigma_r + B - \frac{D}{|\sigma|^2} \right) = 0. \tag{32}$$

Since the modes are oscillatory and unstable, i.e., $\square_i \neq 0$ and $\square_r > 0$, therefore for the consistency of equation (32), we must have

$$B - \frac{D}{|\sigma|^2} < 0 \quad \text{and} \quad 2\sigma_r - \frac{D}{|\sigma|^2} < 0$$

Now $B - \frac{D}{|\sigma|^2} < 0$ gives $|\sigma|^2 < D/B$

or $\sigma_r^2 + \sigma_i^2 < D/B \tag{33}$

The equation (33) is the equation of circular disc of radius $\sqrt{D/B}$ in $\square_r - \square_i$ plane.

Now, if $2\sigma_r - \frac{D}{|\sigma|^2} < 0$

then, $\square_r(|\sigma|^2) < D/2$

or $\sigma_r(\sigma_r^2 + \sigma_i^2) < D/2$

which gives

$$\sigma_r^3 + \sigma_r \sigma_i^2 < D/2. \quad (34)$$

From (33) and (34), we have that the oscillatory unstable modes lie inside the intersection of the circular disc of radius $\sqrt{D/B}$ and the curve $\sigma_r^3 + \sigma_r \sigma_i^2 < D/2$. This intersected area is shown in the following figure.

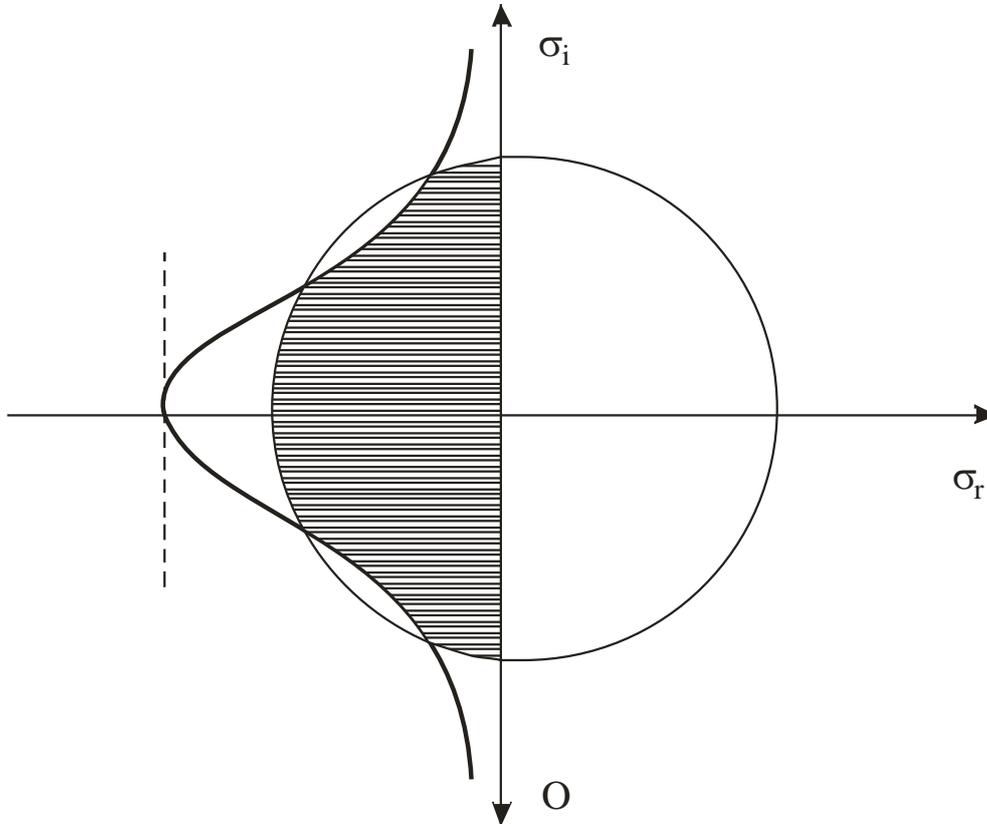


Fig. 2

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